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INTERNATIONAL A LEVEL MATHEMATICS • PURE 3

Pure Mathematics 3

Exam Questions by Topic: Mark Scheme

Pearson Edexcel International Advanced Level (WMA13/01) • Calculator allowed

Every question in the booklet, marked line by line the way an examiner marks it

Chapters (tap a chapter to jump to it)	Qs	Marks	Page	My score
1 Algebraic Methods Algebraic fractions, improper fractions and algebraic division	10	69	own PDF	
2 Functions and Graphs Modulus, composite and inverse functions, transformations	10	76	own PDF	
3 Trigonometric Functions sec, cosec, cot, identities, inverse trigonometric functions	10	72	own PDF	
4 Trigonometric Addition Formulae Addition and double-angle formulae, $R\cos(\theta \pm \alpha)$	10	75	own PDF	
5 Exponentials and Logarithms e^x , $\ln x$, modelling, logarithmic graphs	10	78	own PDF	
6 Differentiation Chain, product and quotient rules, trigonometric and exponential functions	10	76	own PDF	
7 Integration Standard integrals, $f(ax+b)$, trigonometric identities, reverse chain rule	10	77	3	
8 Numerical Methods Locating roots, iteration	10	77	own PDF	

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How to use this mark scheme

The same marking rules Pearson examiners use for WMA13.

Mark every line, not just the answer. Method marks are given for a correct method even when the final answer is wrong. If you lost marks, find the *first* line where your working differs from the scheme. That is the thing to fix.

Code	Meaning
M1	Method mark: for knowing a correct method and attempting to apply it. Small slips in the working do not lose this mark.
dM1	Dependent method mark: can only be given if the method mark before it has been given.
A1	Accuracy mark: for a correct answer or line of working. It can only be given if the method mark it depends on has been given.
B1	Independent mark: for a correct result or statement, whatever method was used.
*	The answer is given in the question. Every step must be shown, and the answer must follow from correct working (cso).
cao	Correct answer only.
cso	Correct solution only: no errors anywhere in the working.
ft	Follow through: the mark can be given for correct work using your own earlier (wrong) answer.
oe	Or equivalent: any correct equivalent form is accepted.

“Solutions relying on calculator technology are not acceptable.” When a question says this, a correct final answer with no algebraic working scores **no marks**. The notes under each scheme line say when this applies.



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Chapter 7: Integration

Mark scheme. Marks are shown on the right of each line.

Question 1

5 marks

Part	Scheme	Marks
(a)	$\frac{3}{2}e^{2x}$ or $-4\ln x $	M1
	$\frac{3}{2}e^{2x} - 4\ln x + c$	A1
		(2)
(b)	$k\ln 2x-3 $	M1
	$\frac{5}{2}\ln 2x-3 + c$	A1
		(2)
(c)	$\frac{1}{4}\tan 4x + c$	B1
		(1)
Total for Question 1: 5 marks		

Question 2

5 marks

Part	Scheme	Marks
(a)	$[x^2 + 3\ln x]_1^e$	M1
	$(e^2 + 3) - (1 + 0)$	M1
	$e^2 + 2$	A1
		(3)
(b)	$[\frac{1}{2}e^{2x}]_0^{\ln 2} = \frac{1}{2}(4 - 1)$	M1
	$\frac{3}{2}$	A1
		(2)
Total for Question 2: 5 marks		



Question 3

5 marks

Part	Scheme	Marks
(a)	$\sin^2 x + \cos^2 x + 2 \sin x \cos x = 1 + \sin 2x$	M1
	Integrates: $x - \frac{1}{2} \cos 2x$	M1
	$x - \frac{1}{2} \cos 2x + c$	A1
		(3)
(b)	$\tan^2 x = \sec^2 x - 1$	M1
	$\tan x - x + c$	A1
		(2)
Total for Question 3: 5 marks		

Question 4

5 marks

Part	Scheme	Marks
(a)	Numerator is the derivative of the denominator: $\ln(3x^2 + 1)$	M1
	$\ln(3x^2 + 1) + c$	A1
		(2)
(b)	$[-\ln(2 + \cos x)]_0^{\frac{\pi}{3}}$	M1
	$-\ln \frac{5}{2} + \ln 3$	M1
	$\ln \frac{6}{5}$	A1
		(3)
Total for Question 4: 5 marks		



Question 5

6 marks

Part	Scheme	Marks
(a)	Divides: quotient $2x - 1$, remainder 2 $2x - 1 + \frac{2}{x+3}$	M1 A1 (2)
(b)	$[x^2 - x + 2\ln(x+3)]_0^2$ Correct integral (4 - 2 + 2ln5) - (0 - 0 + 2ln3) $2 + 2\ln\frac{5}{3}$	M1 A1 M1 A1 (4)
Total for Question 5: 6 marks		

Question 6

10 marks

Part	Scheme	Marks
(a)	Divides by $(x+2)$: $(x+2)^2 = x^2 + 4x + 4$ Remainder 3, so $y = x + 2 + \frac{3}{x+2}$ *	M1 A1* (2)
(b)	$\left[\frac{x^2}{2} + 2x + 3\ln(x+2)\right]_0^2$ Correct integral (2 + 4 + 3ln4) - (3ln2) $6 + 3\ln 2$	M1 A1 M1 A1 (4)
(c)	$\frac{dy}{dx} = 1 - \frac{3}{(x+2)^2}$ (x+2) ² = 3, so $x = -2 + \sqrt{3}$ (as $x > -2$) $y = \sqrt{3} + \frac{3}{\sqrt{3}}$ $(-2 + \sqrt{3}, 2\sqrt{3})$	M1 A1 M1 A1 (4)
Total for Question 6: 10 marks		



Question 7

10 marks

Part	Scheme	Marks
(a)	$4 \cos^2 x - 4 \sin x \cos x + \sin^2 x$ Uses $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$, $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$, $2 \sin x \cos x = \sin 2x$ $2 + 2 \cos 2x - 2 \sin 2x + \frac{1}{2} - \frac{1}{2} \cos 2x$ $\frac{5}{2} + \frac{3}{2} \cos 2x - 2 \sin 2x$	M1 M1 A1 A1 (4)
(b)	$\left[\frac{5}{2}x + \frac{3}{4} \sin 2x + \cos 2x \right]_0^{\frac{\pi}{4}}$ Correct integral $\left(\frac{5\pi}{8} + \frac{3}{4} + 0 \right) - (0 + 0 + 1)$ $\frac{5\pi}{8} - \frac{1}{4}$	M1 A1 M1 A1 (4)
(c)	Replacing x by $2x$ halves the interval and halves the integral $\frac{5\pi}{16} - \frac{1}{8}$	M1 A1 (2)
Total for Question 7: 10 marks		



Question 8

10 marks

Part	Scheme	Marks
(a)	$\frac{\sec x \tan x + \sec^2 x}{\sec x + \tan x}$ (chain rule) Correct derivatives of $\sec x$ and $\tan x$ Numerator = $\sec x(\tan x + \sec x)$ Cancels to $\sec x$ *	M1 A1 M1 A1* (4)
(b)	$\ln(\sqrt{2} + 1) - \ln(1 + 0)$ $\ln(1 + \sqrt{2})$	M1 A1 (2)
(c)	$\int \tan x \, dx = \ln \sec x $ (or $-\ln \cos x $) $\ln 2 - \ln 1 = \ln 2$ *	M1 A1* (2)
(d)	$\sec x > \tan x$ on the interval: $\ln(1 + \sqrt{2}) - \ln \sqrt{2}$ $\ln\left(\frac{1 + \sqrt{2}}{\sqrt{2}}\right) \left(= \ln\left(1 + \frac{\sqrt{2}}{2}\right)\right)$	M1 A1 (2)
Total for Question 8: 10 marks		



Question 9

10 marks

Part	Scheme	Marks
(a)	$[2 \ln(2x+1)]_0^k = 2 \ln(2k+1)$ $(2k+1)^2 = 25$ $k = 2$	M1 M1 A1 (3)
(b)	$\left[\frac{1}{2}e^{2x} + e^x\right]_0^{\ln p}$ $\left(\frac{1}{2}p^2 + p\right) - \left(\frac{1}{2} + 1\right) = 10$ $p^2 + 2p - 23 = 0$ * $p = \frac{-2 \pm \sqrt{96}}{2}$ $p = -1 + 2\sqrt{6}$ (the negative root is rejected as $p > 1$)	M1 M1 A1* M1 A1 (5)
(c)	$[3 \ln(x-1)]_2^3 = 3 \ln 2 - 3 \ln 1$ $= \ln 2^3 = \ln 8$ *	M1 A1* (2)
Total for Question 9: 10 marks		



Question 10

11 marks

Part	Scheme	Marks
(a)	$y = 0: (e^x - 1)(e^x - 2) = 0$ $e^x = 1$ (the origin) or $e^x = 2$ $x = \ln 2$ *	M1 A1 A1* (3)
(b)	$e^{2x} \rightarrow 0$ and $e^x \rightarrow 0$: $y = 2$	B1 (1)
(c)	$\int (e^{2x} - 3e^x + 2) dx = \frac{1}{2}e^{2x} - 3e^x + 2x$ Correct integral $(2 - 6 + 2\ln 2) - (\frac{1}{2} - 3) = -\frac{3}{2} + 2\ln 2$ Region below the axis, so area = $\frac{3}{2} - 2\ln 2$ $\frac{3}{2} - \ln 4$	M1 A1 M1 M1 A1 (5)
(d)	$\frac{dy}{dx} = 2e^{2x} - 3e^x = 8 - 6 = 2$ at $x = \ln 2$ $y = 2(x - \ln 2)$	M1 A1 (2)
Total for Question 10: 11 marks		

