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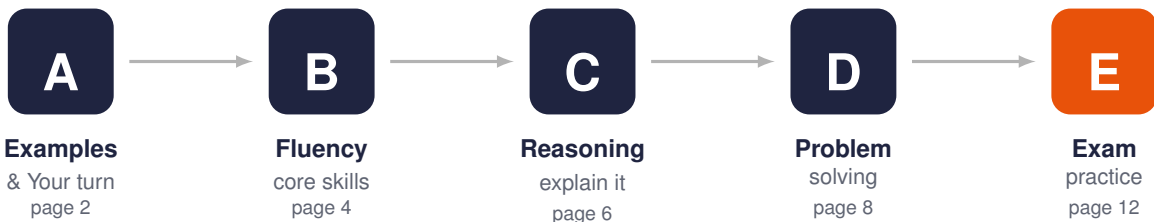
A-LEVEL MATHEMATICS • PURE 1

Graphs and Transformations

Worksheet

Pearson Edexcel International A Level (WMA11) & A Level (9MA0) • Calculator allowed: show every step

Free Graphs and Transformations worksheet for A-Level and International A-Level Maths, with full worked solutions



Tap a section to jump straight to it

By the end of this worksheet you can: sketch cubic curves from their factors, showing where they cross or touch the axes • sketch reciprocal curves and state their asymptotes • find points of intersection and count the solutions of an equation from a sketch • translate and stretch graphs, and find the images of points and asymptotes • *Stretch:* sketch quartics, combine transformations, find a constant from a transformed curve and prove results.

Which questions are for me?

CORE The methods every A-level student must secure. Start here.

STRETCH Quartic curves, reciprocal curves after two transformations, counting solutions with a constant, and reasoning about what a transformation keeps.

Every section has both, clearly labelled. Questions get harder as you go, and every skill in Sections A to D is needed in Section E.

Track your progress

Section	B	C	D	E
Core marks	23	10	13	11
Stretch marks	16	14	17	8
My score				

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A

Worked examples & Your turn

Read each example carefully. Cover it up, then answer the question beside it using the same method.

CORE

Secure the method

Worked example A1

Sketch the curve with equation $y = x^3 - x^2 - 6x$, showing where it meets the coordinate axes.

Take out x first, then factorise the quadratic.

$$x(x^2 - x - 6) = x(x - 3)(x + 2)$$

Positive x^3 : from bottom left to top right. Three different factors: it crosses three times.

crosses at $(-2, 0)$, $(0, 0)$, $(3, 0)$

Your turn A1

Sketch the curve with equation $y = 4x - x^3$, showing where it meets the coordinate axes.

Worked example A2

Sketch $y = \frac{6}{x^2}$, and find where it meets the line $y = 24$

x^2 is never negative, so both branches are above the x -axis. Asymptotes $x = 0$ and $y = 0$

$$\frac{6}{x^2} = 24, \text{ so } x^2 = \frac{1}{4}$$

$\left(-\frac{1}{2}, 24\right), \left(\frac{1}{2}, 24\right)$

Your turn A2

Sketch $y = -\frac{8}{x}$, and find where it meets the line $y = 2$

Worked example A3

Find the points where the curve $y = x(x - 1)(x + 2)$ meets the line $y = 4x$

Never divide by x : you would lose the point at the origin. Bring everything to one side.

$$x(x - 1)(x + 2) - 4x = x(x^2 + x - 6) = x(x + 3)(x - 2)$$

$(0, 0)$, $(2, 8)$, $(-3, -12)$

Your turn A3

Find the points where the curve $y = x(x + 1)(x - 3)$ meets the line $y = 5x$



STRETCH

Worked example A4: a quartic

Sketch $y = (x - 1)^2(x + 2)(x - 3)$, showing where it meets the axes.

Positive x^4 : both ends go up. The squared factor touches; the others cross.

y intercept: $(-1)^2 \times 2 \times (-3) = -6$

touches $(1, 0)$; crosses $(-2, 0)$, $(3, 0)$; $(0, -6)$

Your turn A4

Sketch $y = -x^2(x - 2)(x + 3)$, showing where it meets the axes.

Worked example A5: a translated reciprocal

For $y = \frac{3}{x+2} - 1$, state the asymptotes and the axis intercepts.

$y = \frac{3}{x}$ moved 2 left and 1 down. The asymptotes move with it.

$x = 0$: $y = \frac{3}{2} - 1 = \frac{1}{2}$. $y = 0$: $x + 2 = 3$

$x = -2$, $y = -1$; $(0, \frac{1}{2})$, $(1, 0)$

Your turn A5

For $y = \frac{4}{x-1} + 2$, state the asymptotes and the axis intercepts.

Worked example A6: stretches

$f(x) = x(x - 4)$ has a minimum at $(2, -4)$. Find the axis points and the minimum of $y = f(2x)$ and of $y = 3f(x)$

$f(2x)$: divide x coordinates by 2. $3f(x)$: multiply y coordinates by 3.

$f(2x)$: $(0, 0)$, $(2, 0)$, $\min(1, -4)$; $3f(x)$: $\min(2, -12)$

Your turn A6

$g(x) = (x+1)(x-5)$ has a minimum at $(2, -9)$. Find the axis points of $y = g\left(\frac{x}{3}\right)$, and the turning point of $y = g\left(\frac{x}{3}\right)$ and of $y = -2g(x)$

Worked example A7: images of a point

$P(3, -2)$ lies on $y = f(x)$. Find the image of P on $y = f(x-1) + 4$, $y = -f(2x)$ and $y = f(-x)$

Changes inside the bracket act on x , the opposite way; changes outside act on y .

$(4, 2)$; $\left(\frac{3}{2}, 2\right)$; $(-3, -2)$

Your turn A7

$Q(-4, 6)$ lies on $y = f(x)$. Find the image of Q on $y = f(x+2) - 1$, $y = -f\left(\frac{x}{2}\right)$ and $y = 3f(-x)$



B**Fluency**

Each question is a designed sequence. From one part to the next, ask yourself: what changed, and what did it do to the answer?

CORE*Secure the method*

B1 For each curve, write down the coordinates of the points where it meets the coordinate axes, and say where it touches the x -axis.

(a) $y = (x - 1)(x - 2)(x + 3)$

(b) $y = (x - 1)^2(x + 3)$

(c) $y = (1 - x)(x - 2)(x + 3)$

(d) $y = x(x - 1)^2$

[4 marks]

B2 Factorise fully, then write down the x coordinates of the points where each curve meets the x -axis.

(a) $y = x^3 - 9x$

(b) $y = x^3 + 2x^2 + x$

(c) $y = 2x^3 - 7x^2 + 3x$

(d) $y = 18x - 2x^3$

[4 marks]

B3 For each curve, write down the equations of the asymptotes and the coordinates of any points where it meets the coordinate axes.

(a) $y = \frac{2}{x}$

(b) $y = \frac{2}{x} + 1$

(c) $y = \frac{2}{x+1}$

(d) $y = \frac{2}{x+1} - 4$

[4 marks]

B4 By sketching two suitable graphs on the same axes, state the number of real solutions of each equation.

(a) $x^2 = \frac{4}{x}$

(b) $x^2 - 4 = \frac{3}{x}$

(c) $x(x - 3) = \frac{2}{x}$

(d) $x^2 = \frac{4}{x^2}$

[4 marks]



CORE

continued: transformations

B5 The point $P(2, 5)$ lies on the curve $y = f(x)$. Write down the coordinates of the image of P on the curve with equation

(a) $y = f(x) + 3$

(b) $y = f(x + 3)$

(c) $y = f(3x)$

[3 marks]

B6 The curve $y = f(x)$ has asymptotes $x = 2$ and $y = -1$. Write down the equations of the asymptotes of

(a) $y = f(x) + 3$

(b) $y = f(x - 3)$

(c) $y = 2f(x)$

(d) $y = f\left(\frac{1}{2}x\right)$

[4 marks]

STRETCH

B7 For each curve, write down the coordinates of the points where it meets the coordinate axes, and say where it touches the x -axis.

(a) $y = (x^2 - 1)(x^2 - 4)$

(b) $y = x^2(x^2 - 9)$

(c) $y = (x - 1)^2(x + 2)^2$

(d) $y = (x^2 + 1)(x^2 - 4)$

[4 marks]

B8 $f(x) = x^2 - 4x$. Write each of the following in the form $ax^2 + bx + c$

(a) $f(x + 2)$

(b) $2f(x)$

(c) $f(2x)$

(d) $f(-x)$

[4 marks]

B9 The curve $y = f(x)$ has a minimum point at $(4, -2)$. Write down the coordinates of the turning point of each curve, and say whether it is a maximum or a minimum.

(a) $y = f(x + 4) - 2$

(b) $y = -f(x)$

(c) $y = f(-x)$

(d) $y = 3f\left(\frac{1}{2}x\right)$

[4 marks]

B10 The curve $y = x(x - 3)^2$ has a maximum point at $(1, 4)$ and touches the x -axis at $(3, 0)$. State the number of real solutions of $x(x - 3)^2 = k$ when

(a) $k = 2$

(b) $k = 4$

(c) $k = 5$

(d) $k = -1$

[4 marks]



C**Reasoning**

Show your working and explain your thinking. A correct answer without a reason does not score full marks.

CORE

Secure the method

- C1** Sam sketches the curve $y = (x - 2)^2(x + 1)$ and draws it crossing the x -axis at $(2, 0)$
Explain Sam's mistake, and write down the coordinates of the point where the curve meets the y -axis. [2 marks]
- C2** Lily says: "The graph of $y = f(x - 2)$ is the graph of $y = f(x)$ moved down by 2."
Explain why Lily is wrong. The point $(1, 3)$ lies on $y = f(x)$: write down its image on $y = f(x - 2)$ [2 marks]
- C3** Ben says: "The curve $y = \frac{5}{x+3}$ has asymptotes $x = 3$ and $y = 5$ "
Explain both of Ben's mistakes, and find the coordinates of the point where the curve meets the y -axis. [3 marks]
- C4** Kai finds where the curve $y = x^3 - 4x$ meets the line $y = 5x$. He divides both sides of $x^3 - 4x = 5x$ by x , gets $x^2 = 9$, and gives the points $(3, 15)$ and $(-3, -15)$
Explain why Kai's answer is incomplete, and give every point where the curve meets the line. [3 marks]



STRETCH

- C5** The curve $y = f(x)$ has a maximum point at $(-6, 4)$. Asha says: "The curve $y = f(3x)$ has its maximum at $(-18, 4)$."

Explain Asha's mistake, state the correct maximum point, and write down an equation of a curve that *does* have its maximum at $(-18, 4)$ [3 marks]

- C6** Show that $(x+1)(x^2 - x + 2) = x^3 + x + 2$. Hence prove that the curves $y = x^2 + 1$ and $y = -\frac{2}{x}$ meet at exactly one point, and find its coordinates. [3 marks]

- C7** A cubic curve touches the x -axis at $(2, 0)$, crosses it at $(-1, 0)$ and crosses the y -axis at $(0, 8)$. Tom says its equation is $y = (x+1)(x-2)^2$

Show that Tom is wrong, and find the correct equation. [3 marks]

- C8** $f(x) = x(x-6)$

(a) Show that the line $y = 16$ meets the curve $y = f(x)$ at two points that are 10 units apart.

(2)

(b) Explain why the line $y = 16$ also meets the curve $y = f(x-7)$ at two points that are 10 units apart.

(1)

(c) Find the value of the positive constant a for which the line $y = 16$ meets the curve $y = f(ax)$ at two points that are 4 units apart.

(2)

[5 marks]



D**Problem solving**

You decide the method. Read the whole question first, and show every step.

CORE*Secure the method*

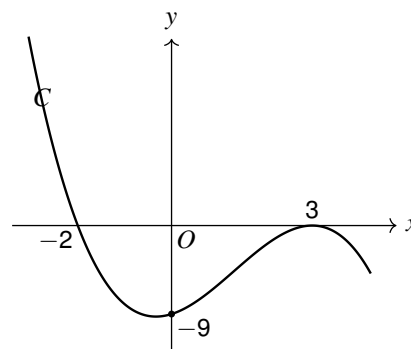
- D1** Figure 1 shows a sketch of the curve C with equation $y = f(x)$, where $f(x)$ is a cubic expression.

The curve C crosses the x -axis at $(-2, 0)$, touches the x -axis at $(3, 0)$ and crosses the y -axis at $(0, -9)$

(a) Find $f(x)$, giving your answer in the form $k(x+p)(x+q)^2$, where k , p and q are constants to be found.

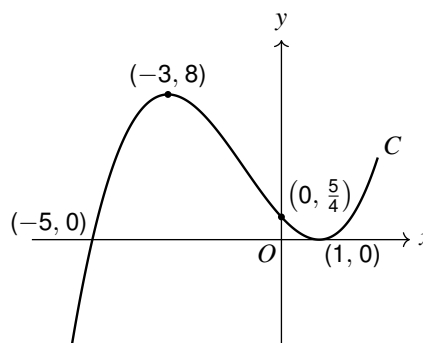
(2)

(b) Using Figure 1, write down the set of values of x for which $f(x) > 0$

(1)**Figure 1****[3 marks]**

- D2** Figure 2 shows a sketch of the curve C with equation $y = f(x)$

The curve crosses the x -axis at $(-5, 0)$, touches the x -axis at $(1, 0)$, crosses the y -axis at $(0, \frac{5}{4})$ and has a maximum point at $(-3, 8)$

**Figure 2**

(a) For the curve with equation $y = f(x+2)$, write down the coordinates of the points where it meets the x -axis and the coordinates of its maximum point.

(2)

(b) For the curve with equation $y = f(2x)$, write down the coordinates of the points where it meets the coordinate axes and the coordinates of its maximum point.

(2)**[4 marks]**

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D3 In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.

Figure 3 shows the curve C with equation $y = \frac{3}{x-2}$, $x \neq 2$, and the line l with equation $y = x - 4$.

The line l meets C at the points A and B .

(a) Write down the equations of the asymptotes of C .

(1)

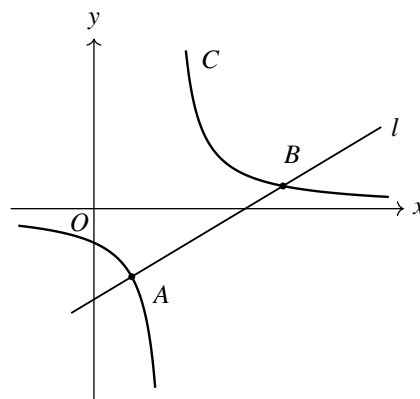


Figure 3

(b) Use algebra to find the coordinates of A and the coordinates of B .

(2)

[3 marks]

D4 Figure 4 shows a sketch of the curve C with equation $y = x^2(3 - x)$.

The curve touches the x -axis at O , crosses it at $(3, 0)$ and has a maximum point at $(2, 4)$.

Giving a reason for each answer, state the number of real solutions of

(a) $x^2(3 - x) = 2$

(1)

(b) $x^2(3 - x) = 4$

(1)

(c) $(x + 1)^2(2 - x) = 2$

(1)

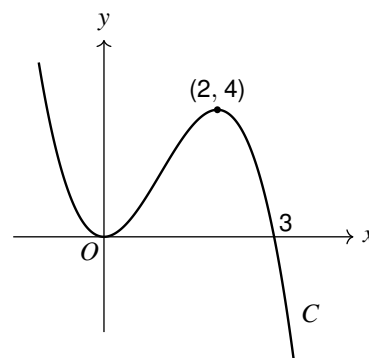


Figure 4

[3 marks]



STRETCH

- D5** In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.

The curve C has equation $y = f(x)$, where

$$f(x) = \frac{2}{x+1} + 3 \quad x \neq -1$$

- (a) Write down the equations of the asymptotes of C , and find the coordinates of the points where C meets the coordinate axes.

(2)

The curve C is transformed to the curve with equation $y = f(x-2) - 3$

- (b) Write the equation of the transformed curve in the form $y = \frac{a}{x+b}$, where a and b are integers, and state the equations of its asymptotes.

(2)

[4 marks]

- D6** In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.

Figure 5 shows the curve C with equation

$$y = x^2(x-2)^2$$

The curve touches the x -axis at O and at $(2, 0)$, and has a maximum point at $(1, 1)$

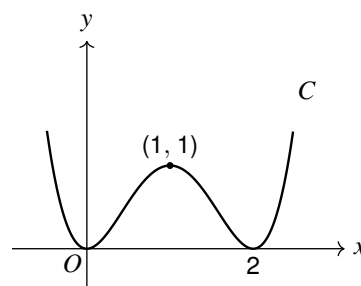


Figure 5

- (a) State the number of real solutions of

$$x^2(x-2)^2 = \frac{1}{2}, \text{ giving a reason.}$$

(1)

- (b) Use algebra to find the exact x coordinates of the points where C meets the line $y = 1$

(2)

- (c) Find the set of values of k for which the equation $x^2(x-2)^2 = k$ has exactly four real solutions.

(1)

[4 marks]



- D7** Figure 6 shows a sketch of the curve C with equation $y = f(x)$, where

$$f(x) = (x - 1)(x - 5)$$

The curve crosses the x -axis at $(1, 0)$ and $(5, 0)$ and has a minimum point at $(3, -4)$

- (a) Find the two values of the constant b for which the curve with equation $y = f(x + b)$ passes through the origin.

(2)

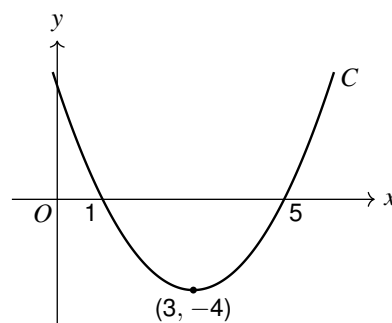


Figure 6

The curve with equation $y = af(x)$, where a is a constant, has a turning point at $(3, 10)$

- (b) Find the value of a , and state whether $(3, 10)$ is a maximum point or a minimum point.

(2)

[4 marks]

- D8** In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.

Figure 7 shows the curve C with equation $y = (x + 1)(x - 2)(x - 4)$ and the straight line l

The line l passes through the point $A(-1, 0)$ and through the point B on C whose x coordinate is 3

- (a) Find an equation for l .

(1)

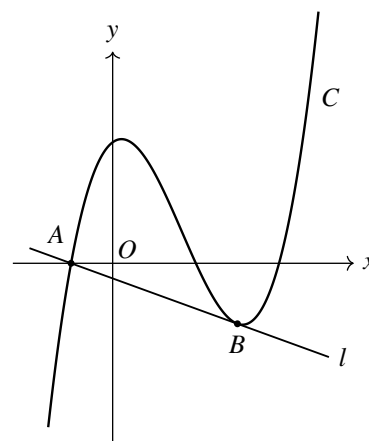


Figure 7

- (b) Show that the x coordinates of the points where l meets C satisfy the equation $(x + 1)(x - 3)^2 = 0$

(3)

- (c) Hence explain what the factor $(x - 3)^2$ tells you about l and C at B .

(1)

[5 marks]



E**Exam practice**

Exam-style questions in the style of WMA11. Write your answers in the spaces, and show every step.

CORE

Exam questions for everyone

- E1** Figure 8 shows a sketch of the curve C with equation $y = f(x)$

The curve C crosses the x -axis at the point $(-4, 0)$, touches the x -axis at the point $(2, 0)$, crosses the y -axis at the point $(0, 4)$ and has a maximum point at $(-2, 8)$

On separate diagrams, sketch the curve with equation

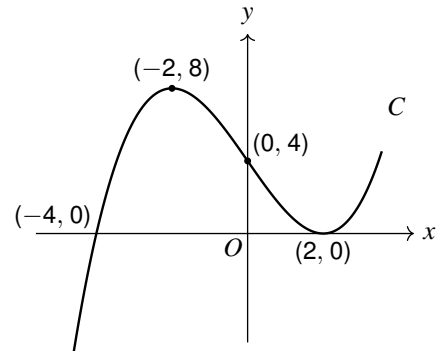


Figure 8

(a) $y = f(x - 2)$

(3)

(b) $y = f(2x)$

(2)

On each diagram, show clearly the coordinates of the maximum point and the coordinates of every point where the curve cuts or touches the coordinate axes.

(Total for Question E1 is 5 marks)



E2 *In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.*

The curve C has equation

$$y = 2 + \frac{3}{x} \quad x \neq 0$$

(a) Sketch the graph of C . On your sketch, show clearly the coordinates of any point where C meets the coordinate axes, and state the equations of the asymptotes of C .

(3)

The straight line l has equation $y = 5 - 2x$

(b) Use algebra to show that the line l does not meet the curve C at any point.

(3)

(Total for Question E2 is 6 marks)



STRETCH

- E3** In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.

Figure 9 shows the curve C_1 with equation $y = \frac{1}{x^2}$,

$x \neq 0$, and the curve C_2 with equation $y = \frac{4}{(x+1)^2}$,

$x \neq -1$

- (a) Describe fully the single transformation that maps the curve with equation $y = \frac{1}{x^2}$ onto the curve with equation $y = \frac{4}{x^2}$. There are two possible answers; give both.

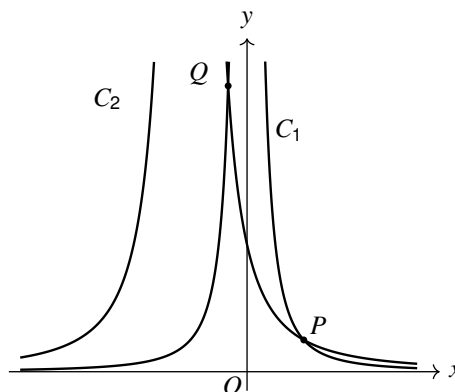


Figure 9

- (b) Describe fully the single transformation that maps the curve with equation $y = \frac{4}{x^2}$ onto C_2 .

.....
(1)

The curves C_1 and C_2 meet at the points P and Q , as shown in Figure 9.

- (c) Use algebra to find the coordinates of P and the coordinates of Q .

.....
(5)

(Total for Question E3 is 8 marks)

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