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Algebraic Fractions

Worked Solutions

Pearson Edexcel GCSE (1MA1) & International GCSE (4MA1) • No calculator

Full worked solutions for the free Algebraic Fractions worksheet, GCSE and iGCSE Maths



Tap a section to jump straight to it

Full worked solutions for every question on the Algebraic Fractions worksheet, in the same order and with the same question numbers.

How to mark your work. Tick each step you got right, not just the final answer. In the exam, method marks are given for correct working even when the final answer is wrong. If you lost marks, find the *first* line where your working is different from ours. That is the thing to fix.

In Section E: **M** = method mark, **P** = process mark, **A** = accuracy mark, **B** = independent mark, **C** = communication mark.

Question sheet

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A**Your turn answers**

Compare each line of your working with the solution.

FOUNDATION

A1 $18 \div 6 = 3$ and $x^3 \div x = x^2$ $3x^2$

A2 $\frac{6x}{10} - \frac{5x}{10} = \frac{x}{10}$

A3 $\frac{3(2x-1) - 4(x+3)}{12} = \frac{6x-3-4x-12}{12} = \frac{2x-15}{12}$

HIGHER

A4 $\frac{2(x+2)(x-2)}{(x+3)(x-2)} = \frac{2(x+2)}{x+3}$

A5 $\frac{4(x+2) - (x-3)}{(x-3)(x+2)} = \frac{4x+8-x+3}{(x-3)(x+2)} = \frac{3x+11}{(x-3)(x+2)}$

A6 $\frac{x(x+3)}{x-2} \times \frac{x-2}{(x+3)(x-3)} = \frac{x}{x-3}$

A7 Multiply by $x(x-1)$: $4x - 3(x-1) = x(x-1)$ $x+3 = x^2 - x$ $x^2 - 2x - 3 = 0$ $(x-3)(x+1) = 0$
 $x = 3$ or $x = -1$

B**Fluency solutions**One mark for each correct part. The notes in *italics* explain why each sequence was chosen.**FOUNDATION**

B1 (a) $2x$ (b) 2 (c) $2x$ (d) $\frac{2}{x}$

B2 (a) $\frac{4(x+2)}{4} = x+2$ (b) $2x-3$ (c) $\frac{5(x+2)}{x+2} = 5$ (d) $\frac{x(x+3)}{x} = x+3$

Only whole factors cancel, so factorise the top first.

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B3 (a) $\frac{5x}{6}$ (b) $\frac{x}{6}$ (c) $\frac{9x}{12} + \frac{2x}{12} = \frac{11x}{12}$ (d) $\frac{3}{x}$

B4 (a) $\frac{2x+2+x+3}{4} = \frac{3x+5}{4}$ (b) $\frac{2x+2-x-3}{4} = \frac{x-1}{4}$

(c) $\frac{5(2x-1)+3(x+2)}{15} = \frac{13x+1}{15}$ (d) $\frac{x+4-2(x-2)}{6} = \frac{8-x}{6}$

second numerator.

The minus sign applies to the whole

B5 (a) 2 (b) $\frac{20x}{5x^2} = \frac{4}{x}$ (c) $\frac{x}{4} \times \frac{2}{x} = \frac{1}{2}$

B6 (a) **yes** (b) **no**: it is $x+1$ (c) **yes**: $\frac{3(x+2)}{6} = \frac{x+2}{2}$ (d) **no**

HIGHER

B7 (a) $x-3$ (b) $\frac{x(x+2)}{(x+2)(x-2)} = \frac{x}{x-2}$ (c) $\frac{(x-3)(x+2)}{(x-3)(x+3)} = \frac{x+2}{x+3}$ (d) $\frac{(2x-1)(x+3)}{(2x-1)(2x+1)} = \frac{x+3}{2x+1}$

B8 (a) $\frac{2x+1}{x(x+1)}$ (b) $\frac{2(x+3)-(x-3)}{x^2-9} = \frac{x+9}{x^2-9}$ (c) $\frac{x+3}{x+2}$ (d) $\frac{x(x+1)-(x-1)}{x^2-1} = \frac{x^2+1}{x^2-1}$

B9 (a) $\frac{x+2}{3(x+2)} = \frac{1}{3}$ (b) $\frac{(x+2)(x-2)}{x} \times \frac{x^2}{x+2} = x(x-2)$

(c) $\frac{x(x+1)}{(x+1)(x-1)} \times \frac{x-1}{x} = 1$ (d) $\frac{2(x+1)}{(x+3)(x-3)} \times \frac{x+3}{x+1} = \frac{2}{x-3}$

B10 (a) $x=2$ (b) $6 = x^2 + x$, $(x+3)(x-2) = 0$: $x=2$ or $x=-3$

(c) $12(x+1)+12x = 7x(x+1)$, $7x^2 - 17x - 12 = 0$, $(7x+4)(x-3) = 0$: $x=3$ or $x = -\frac{4}{7}$

(d) $x+3 = x(x-1)$, $x^2 - 2x - 3 = 0$: $x=3$ or $x=-1$



C

Reasoning solutions

Your wording can be different. Check that your reason makes the same point.

FOUNDATION

- C1** Kim cancelled the 6 in one term only. You can only cancel a factor of the whole top.

$$\frac{x}{6} + 1$$

- C2** Tom added the numerators and the denominators. Use a common denominator of 6:

$$\frac{5x}{6}$$

C3 $\frac{3(x+1)+(x+5)}{6} = \frac{4x+8}{6} = \frac{2(2x+4)}{6} = \frac{2x+4}{3}$

as required

- C4** (a) $3+2 = 5$ (b) $\frac{30}{6} = 5$ (c) $\frac{5x}{6}$ is the same expression as $\frac{x}{2} + \frac{x}{3}$, so they are equal for every x

HIGHER

- C5** The x^2 terms are not factors of the whole top and bottom, so they cannot be cancelled. $x^2 + 9$ does not factorise, so the fraction **cannot be simplified**.

- C6** $\frac{(x+2)(x-2)}{x-2} = x+2$ after cancelling $(x-2)$. When $x = 2$ the denominator is 0, so the fraction is not defined there.

C7 $\frac{(x+1)-(x-1)}{(x-1)(x+1)} = \frac{2}{x^2-1}$

as required

C8 (a) $x + \frac{1}{x} - 2 = \frac{x^2+1-2x}{x} = \frac{(x-1)^2}{x}$

as required

(b) $(x-1)^2 \geq 0$ and $x > 0$, so $\frac{(x-1)^2}{x} \geq 0$. Therefore $x + \frac{1}{x} \geq 2$, with equality only when **$x = 1$**



D**Problem solving solutions**

There is often more than one correct method. Any complete, correct method scores full marks.

FOUNDATION

D1 $2\left(\frac{x}{3} + \frac{x}{4}\right) = \frac{7x}{6}$ $\frac{7x}{6} = 7$ $x = 6$

D2 (a) $\frac{x}{2} + \frac{x}{3} + \frac{x}{6} = \frac{3x+2x+x}{6} = x$, and $x \div 3 = \frac{x}{3}$ **as required** (b) $\frac{x}{3} = 5$: $x = 15$

D3 $\frac{x}{3} + \frac{x}{4} = \frac{7x}{12}$ and $\frac{x}{2} + \frac{x}{5} = \frac{7x}{10}$. Tenths are bigger than twelfths, so $\frac{x}{2} + \frac{x}{5}$ is bigger

D4 $\frac{6(x+2)}{3(x+2)} = 2$. The bracket cancels for every x except $x = -2$, where the denominator is 0

HIGHER

D5 $\frac{24}{x} + \frac{24}{x+4} = 5$ $24(x+4) + 24x = 5x(x+4)$ $5x^2 - 28x - 96 = 0$ $(5x+12)(x-8) = 0$

A speed must be positive: $x = 8$ km/h Check: $3 + 2 = 5$ hours

D6 In 1 hour A fills $\frac{1}{x}$ and B fills $\frac{1}{x+3}$; together they fill $\frac{1}{2}$ **as required**

$2(x+3) + 2x = x(x+3)$ $x^2 - x - 6 = 0$ $(x-3)(x+2) = 0$ $x = 3$

D7 $\frac{x(x+2)(x-2)}{(x+2)^2} = \frac{x(x-2)}{x+2}$

D8 (a) Multiply by $x^2 - 4$: $(x+2) + (x-2) = 4$ $2x = 4$ $x = 2$

(b) When $x = 2$ the denominators $x - 2$ and $x^2 - 4$ are 0, so $x = 2$ is not allowed. There is **no solution**.



E**Exam practice solutions**

Marks are shown on the right of each line.

FOUNDATION*Foundation and Higher crossover*

E1 (a) $3x$ B1
 (b) $2x + 3$ B1

E2 $\frac{5x}{15} + \frac{3x}{15}$ M1
 $\frac{8x}{15}$ A1

E3 $\frac{3(x+1)+2(x-2)}{12}$ M1
 $\frac{3x+3+2x-4}{12}$ M1
 $\frac{5x-1}{12}$ A1

E4 (a) $\frac{18x}{12x^2}$ M1
 $\frac{3}{2x}$ A1
 (b) $\frac{x}{4} \times \frac{6}{x} = \frac{3}{2}$ B1

E5 Ben divided only the $2x$ by 2; the 6 must be divided too C1
 $x+3$ B1

E6 $\frac{2(2x+1)-3(x-4)}{6}$ M1
 $\frac{4x+2-3x+12}{6}$ M1
 $\frac{x+14}{6}$ A1



HIGHER

E7 $\frac{(x+3)(x-3)}{(x+3)(x-2)}$ M1 M1

$$\frac{x-3}{x-2}$$
A1

E8 $\frac{3(x-1)-2(x+2)}{(x+2)(x-1)}$ M1

$$\frac{3x-3-2x-4}{(x+2)(x-1)}$$
M1

$$\frac{x-7}{(x+2)(x-1)}$$
A1

E9 Multiply by $(x-1)(x+1)$: $3(x+1)+2(x-1)=2(x^2-1)$ M1

$$5x+1=2x^2-2, \text{ so } 2x^2-5x-3=0$$
M1

$$(2x+1)(x-3)=0$$
M1

$$x=3 \text{ or } x=-\frac{1}{2}$$
A1

E10 (a) $\frac{(x+2)(x-2)}{(x+2)(x+3)}$ M1

$$\times \frac{2(x+3)}{x-2}$$
M1

Cancel $(x+2)$, $(x-2)$ and $(x+3)$ M1

$= 2$ as required A1

(b) The denominators x^2+5x+6 and $x-2$ are zero when $x = -2, -3 \text{ or } 2$ B1

