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A-LEVEL MATHEMATICS • DECISION 1

Graphs and Networks

Worked Solutions

Pearson Edexcel International A Level (WDM11) • Calculator allowed: show every step

Full worked solutions for the free Graphs and Networks worksheet, A-Level and International A-Level Maths

A — **Examples** & Your turn — page 2

B — **Fluency** core skills — page 2

C — **Reasoning** explain it — page 4

D — **Problem** solving — page 4

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Tap a section to jump straight to it

Full worked solutions for every question on the Decision 1 Graphs and Networks worksheet, in the same order and with the same question numbers.

How to mark your work. Tick each step you got right, not just the final answer. At A-level most marks are for method, so a wrong final answer can still earn most of the marks. If you lost marks, find the *first* line where your working is different from ours. That is the thing to fix.

In Section E: **M** = method mark, **dM** = method mark that depends on the previous M mark, **A** = accuracy mark (needs the M mark before it), **B** = independent mark.

Question sheet | Exam questions by topic | Decision 1 exam questions | More worksheets

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A Your turn answers

Compare each line of your working with the solution.

CORE

A1 P3, Q3, R3, S3, T2

A2 $\frac{16}{2}$: 8 arcs

A3 PQ, QR, RS, ST all exist and no vertex repeats: a path

STRETCH

A4 $\frac{1}{2} \times 9 \times 8$ and $15 - 1$: 36 and 14

A5 3×6: 18

A6 Sum of entries $14 = 2\times$ arcs: 7

A7 $7+4+9+2+6+5+8$: 41

B Fluency solutions

One mark for each correct part. The notes in italics explain why each sequence was chosen.

CORE

B1 **(a)** A3, B4, C3, D3, E4, F3 **(b)** $\frac{20}{2}$: 10 **(c)** 4 **(d)** simple and connected

The number of odd vertices is always even.

B2 **(a)** e.g. A–D–E–B–C **(b)** yes: closed, every arc exists, no vertex repeated **(c)** a trail: B repeats, no arc repeats **(d)** A–B–C–D–E–F–A

Path: no repeated vertex. Trail: no repeated arc.

B3 **(a)** **45** **(b)** **24** **(c)** **11** **(d)** $\frac{1}{2}n(n-1) = 66$: **$n = 12$**

Three formulae, then one used backwards.

B4 **(a)** $\frac{18}{2}$: **9** **(b)** the sum is 13, which is odd **(c)** a degree of 5 needs 5 other vertices but there are only 4 **(d)** 2×5: **10**

Parity, then the simple-graph limit.

B5 **(a)** **A3, B4, C3, D2** (the loop counts 2) **(b)** $\frac{12}{2}$: **6** **(c)** **not simple** (a loop at B), **connected**

A loop is recorded as 2 on the diagonal.

B6 **(a)** **7** **(b)** **44** **(c)** **A3, B3, C3, D3, E2** **(d)** A–B–D 13, A–C–D 14, A–E–D 14: **A–B–D, 13**

A dash means no direct arc.

STRETCH

B7 **(a)** G_1: A3, B2, C3, D2; G_2: P2, Q3, R2, S3 **(b)** **A→Q, B→P, C→S, D→R**

(c) K_4 has 6 arcs, so it is K_4 with one arc removed, and every arc of K_4 is alike **(d)** **6**

A correspondence must preserve every arc.

B8 **(a)** **7** **(b)** $2+3+2=7$ and $2+2+1+2=7$ **(c)** $12-7$: **5** **(d)** it joins two vertices in the same set

Every arc has one end in each set.

B9 **(a)** **8** **(b)** **5** **(c)** an arc of a cycle could be removed and the tree would stay connected, so it would not be minimal **(d)** 2×8: **16**

A tree on n vertices has $n-1$ arcs.

B10 **(a)** **7** **(b)** **28** **(c)** six vertices of degree 7 and two of degree 6 **(d)** every pair is joined, so the vertices can be visited in any order

Between a tree and K_8

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C Reasoning solutions

Your wording can be different. Check that your reason makes the same point.

CORE

C1 A path may not repeat a vertex; A appears twice, so it is **a trail**.

C2 The degree sum would be 9, which is odd, but it must be twice the number of arcs.

C3 He did not halve: each arc adds 2 to the sum, so $\frac{14}{2}$: **7 arcs**

C4 A loop adds 2 to the degree: **degree 4**; it appears as **2** on the diagonal.

STRETCH

C5 A 6-cycle and two separate triangles both have every degree 2, but one is connected and the other is not, so they are not isomorphic.

C6 A tree with n vertices has $n-1$ arcs: **7**; an eighth arc would create a cycle.

C7 Each of the 3 vertices joins each of the other 3: $3 \times 3 = 9$

C8 **(a)** each arc has two ends, so it adds 1 to the degree of each, 2 to the sum; the sum is twice the number of arcs, as required

(b) the sum is even; the even degrees have an even total, so the odd degrees have an even total, which needs an even number of them, as required

D Problem solving solutions

There is often more than one correct method. Any complete, correct method scores full marks.

CORE

D1 **(a)** **P2, Q3, R3, S3, T3, U2**; $\frac{16}{2}$: **8 arcs**

(b) the sum $1+2+2+3+3+4=15$ is odd, but it must be twice the number of roads

(c) K_6 has 15 arcs; $15-8$: **7**

D2 **(a)** **A–F–C–D**

(b) **yes**: AB, BF, FD, DE, EA all exist, it is closed and no vertex repeats

(c) **A–F–B–C–D–E–A**

(d) degrees 3, 3, 3, 3, 2, 4 sum to 18: **9 arcs**

D3 **(a)** rows A: 0 2 1 0; B: 2 0 0 1; C: 1 0 0 1; D: 0 1 1 2

(b) **12** (2×6 arcs); not simple: a double arc AB and a loop at D

D4 **(a)** **77 m**

(b) 11 arcs, a tree on 8 vertices has 7: **4**

(c) closed with no repeated vertex, so **a cycle**: $7+12+8+6+3=$ **36 m**

STRETCH

D5 **(a)** **A→Q, B→P, C→R, D→U, E→S, F→T**; each arc of Figure 6 maps to an arc of Figure 7

(b) Figure 6 has a triangle (A, B, C) but Figure 8 has none, so no correspondence can preserve the arcs

D6 **(a)** every arc joins a student to a club: **8 arcs**

(b) students $2+3+3=8$; clubs $2+1+2+1+2=8$

(c) $15-8$: **7**

(d) W and X are both students, so the arc would lie within one set

D7 **(a)** **8 arcs, 52 km**

(b) **A2, B3, C3, D3, E3, F2**: **4** of odd degree

(c) A is joined only to B and C, and neither is joined to F, so at least **3 roads**; A–B–D–F uses 3

D8 **(a)** **A2, B3, C3, D2, E3, F3, G4**; total weight **105**

(b) A–B–C–D–E–G–F–A: 79; A–B–G–C–D–E–F–A: 74; no others: **shortest 74 minutes**

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E Exam practice solutions

Mark-scheme notes are shown on the right of each line.

CORE *Exam questions for everyone*

E1 **(a)** 11 arcs; degrees sum to $22 = 2 \times 11$ *B1*

217 km *B1*

(b) (i) a path (no vertex repeated); (ii) a trail (B repeated, no arc repeated); (iii) a cycle (closed, no vertex repeated) *B1 B1*

(c) D and H have degree 2, so CD, DE, GH, HA are forced *M1*

if AE is used, AB and EF are not, F needs FG and FC, then BC is not used and B has only one arc: impossible *A1*

so AB, EF are used; BG would close A–B–G–H–A early, so BC and FG are used *A1*

A–B–C–D–E–F–G–H–A, 144 km *A1*

(d) **7 arcs**; **4 roads** unused *B1 B1*

E2 **(a)** network drawn with the 10 arcs and their distances *M1 A1 A1*

(b) **10 arcs, 85 miles** *B1*

(c) **J2, K3, L3, M3, N4, P3, R2** *B1*

the sum 20 is twice the 10 arcs *B1*

(d) **J–K–M–P–R–N–L–J, 68 miles** *M1 A1*

J and R have degree 2, and a Hamiltonian cycle must enter and leave each vertex *B1*

(e) K_7 has 21 arcs; $21 - 10$: **11** *B1*

(f) a tree on 7 vertices has 6 arcs; there are 10 (and cycles such as J–K–L–J) *B1*

4 roads *B1*

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STRETCH

E3 **(a)** every vertex has degree 3 *B1*

$\frac{24}{2}$: **12 arcs** *B1*

(b) both graphs: 8 vertices, 12 arcs, all degrees 3 *B1*

A→1, B→2, C→3, D→8, E→6, F→5, G→4, H→7 *M1 A1*

each of the 12 arcs of G maps to an arc of H (e.g. AE→16, CG→34) *A1*

(c) **{A, C, F, H} and {B, D, E, G}** *M1 A1*

every arc of G joins a vertex of one set to a vertex of the other *A1*

(d) $K_{4,4}$ has 16 arcs; $16 - 12$ *M1*

4 *A1*

(e) **A–B–C–D–H–G–F–E–A** *B1*

(f) a tree on 8 vertices has 7 arcs *M1*

G has 12, so it contains cycles and is not a tree *A1*

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