eClassroom

A-LEVEL MATHEMATICS • DECISION 1

Linear Programming

Worked Solutions

Pearson Edexcel International A Level (WDM11) • Calculator allowed: show every step

Full worked solutions for the free Linear Programming worksheet, A-Level and International A-Level Maths

A **Examples** & Your turn page 2

B **Fluency** core skills page 2

C **Reasoning** explain it page 3

D **Problem** solving page 4

E **Exam** practice page 5

Tap a section to jump straight to it

Full worked solutions for every question on the Decision 1 Linear Programming worksheet, in the same order and with the same question numbers.

How to mark your work. Tick each step you got right, not just the final answer. At A-level most marks are for method, so a wrong final answer can still earn most of the marks. If you lost marks, find the *first* line where your working is different from ours. That is the thing to fix.

In Section E: **M** = method mark, **dM** = method mark that depends on the previous M mark, **A** = accuracy mark (needs the M mark before it), **B** = independent mark.

Question sheet | Exam questions by topic | Decision 1 exam questions | More worksheets

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A Your turn answers

Compare each line of your working with the solution.

CORE

A1 $\boxed{y \leqslant 3x}$

A2 $y \leqslant 0.4(x+y)$, so $0.6y \leqslant 0.4x$: $\boxed{3y \leqslant 2x}$

A3 $\frac{x}{150} + \frac{y}{250} \leqslant 1$; multiply by 750: $\boxed{5x+3y \leqslant 750}$

STRETCH

A4 $x = 16 - 4y$; $48 - 12y + 2y = 18$: $\boxed{(4,\ 3)}$

A5 P: 0, 12, 17, 12: $\boxed{\textbf{17 at } (4,3)}$

A6 $\boxed{-\frac{4}{5}}$

A7 $\boxed{(5,1),\ (6,1),\ (5,2),\ (6,2)}$, if in R

B Fluency solutions

One mark for each correct part. The notes in italics explain why each sequence was chosen.

CORE

B1 **(a)** x and y are the numbers of litres of soup and of stew made each day **(b)** $\boxed{\textbf{Maximise } P = 4x+5y}$

(c) $\boxed{2x+3y \leqslant 60}$ **(d)** $\boxed{y \geqslant 5}$

Variables need units and a time period.

B2 **(a)** $\boxed{(\frac{11}{2},\ \frac{9}{2})}$ **(b)** $\boxed{(3,\ 7)}$ **(c)** $(0,0)$, $(7,0)$, $(\frac{11}{2}, \frac{9}{2})$, $(3,7)$, $(0,8)$ **(d)** $\boxed{\textbf{yes}}$: $10 \leqslant 10$, $20 \leqslant 21$, $20 \leqslant 24$

Solve each pair of boundary lines exactly.

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B3 **(a)** 0, 28, 44.5, 47, 40 **(b)** **47 at** $(3,7)$ **(c)** $(\frac{11}{2},\frac{9}{2})$ **(d)** $\frac{73}{2}$

A new objective can move the optimum.

B4 **(a)** $(0,11)$, $(3,5)$, $(\frac{20}{3},\frac{4}{3})$, $(12,0)$ **(b)** $\frac{80}{3}$ **at** $(\frac{20}{3},\frac{4}{3})$ **(c)** **8** **(d)** $x+y$ is parallel to the edge $x+y=8$, so every point from $(3,5)$ to $(\frac{20}{3},\frac{4}{3})$ is optimal

Minima on open regions use the vertex method too.

B5 **(a)** $-\frac{2}{5}$ **(b)** away from the origin **(c)** $(3,7)$**, 41**

The last point the line touches.

B6 **(a)** $(\frac{77}{19},\frac{68}{19})$ **(b)** $\frac{648}{19}$ **(c)** $(4,3)$: 31; $(3,4)$: 32; $(4,4)$: not in R **(d)** $(3,4)$**, 32**

Test points near the vertex, inside R.

STRETCH

B7 **(a)** $3x \geqslant 2y$ **(b)** $y \leqslant 3x$ **(c)** $y \geqslant x+10$ **(d)** $x+y \geqslant 50$

Percentages compare with the total.

B8 **(a)** $2x+3y=18$ **(b)** $y=2x$ **(c)** $2x+3y \leqslant 18$, $y \leqslant 2x$, $y \geqslant 1$ **(d)** $(\frac{1}{2},1)$, $(\frac{9}{4},\frac{9}{2})$, $(\frac{15}{2},1)$

Test a point to choose each side.

B9 **(a)** -1 **and** $-\frac{1}{3}$ **(b)** $-\frac{1}{k}$ **(c)** $-1 \leqslant -\frac{1}{k} \leqslant -\frac{1}{3}$: $1 \leqslant k \leqslant 3$ **(d)** $(0,8)$

The objective must lie between the edges at the vertex.

B10 **(a)** $3x+2y \leqslant 180$ **(b)** $x+3y \leqslant 120$ **(c)** $(\frac{300}{7},\frac{180}{7})$ **(d)** both machines are used for the whole day

Fractions of a day add to at most 1.

C Reasoning solutions

Your wording can be different. Check that your reason makes the same point.

CORE

C1 The objective must say what to do: **Maximise** $P=4x+5y$.

C2 They must say what is counted, with units and time: the number of litres of soup (stew) made each day.

C3 30% is of the total, $x+y$: $x \geqslant 0.3(x+y)$: $\boxed{7x \geqslant 3y}$

C4 $(4,4)$ breaks $5x+3y \leqslant 31$ ($32 > 31$); test points inside R: $\boxed{(3,4)\text{, } \mathbf{32}}$

STRETCH

C5 For a minimum the line is moved towards the origin and the first point of R reached is optimal: $\boxed{(\frac{20}{3}, \frac{4}{3})}$

C6 Vertices must be found exactly by solving simultaneously: $\boxed{(\frac{11}{2}, \frac{9}{2})}$

C7 Shading shows the unwanted side; for $y \geqslant 2$ the region below $y = 2$ is shaded and R lies above.

C8 **(a)** the objective is linear, so along any edge it increases in one direction (or stays constant); a maximum at an interior or edge point can be moved along an edge to a vertex without decreasing P

(b) substitute a point not on the line, such as the origin: if it satisfies $ax+by \leqslant c$, that side is wanted

D Problem solving solutions

There is often more than one correct method. Any complete, correct method scores full marks.

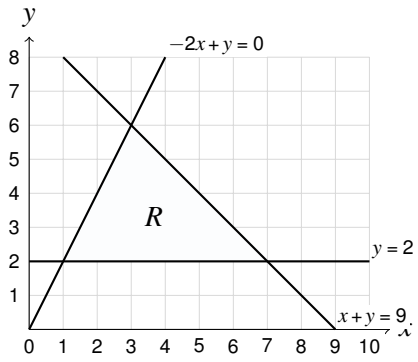
CORE

D1 $\frac{x}{20}+\frac{y}{50} \leqslant 1$: $\boxed{5x+2y \leqslant 100}$; $\frac{x}{30}+\frac{y}{40} \leqslant 1$: $\boxed{4x+3y \leqslant 120}$; $\boxed{y \geqslant 2x}$

D2 $\boxed{2x+3y \leqslant 30}$, $\boxed{y \leqslant x+2}$, $\boxed{x \leqslant 12}$, $\boxed{y \geqslant 1}$

D3 Vertices $(0,7)$: 35; $(\frac{8}{3},7)$: 43; $(\frac{44}{7}, \frac{30}{7})$: $\frac{282}{7}$; $(8,0)$: 24: $\boxed{\mathbf{43 \text{ at } (\frac{8}{3}, 7)}}$

D4



Solution to D4

vertices $\boxed{(1,2),\ (3,6),\ (7,2)}$

STRETCH

D5 **(a)** last vertex reached is where $x+2y=20$ meets $y=x+2$: $\boxed{(\frac{16}{3},\frac{22}{3}),\ P=\frac{82}{3}}$

(b) moving towards the origin, the first point reached is $(4,0)$: $\boxed{P=4}$

D6 **(a)** x, y: the numbers of trays of herbs and of flowers sold each week; $\boxed{\textbf{Maximise } P=3x+4y}$

(b) $\boxed{x+y\leqslant 200}$, $\boxed{y\geqslant 30}$, $x\leqslant 0.6(x+y)$: $\boxed{2x\leqslant 3y}$, $x,\ y\geqslant 0$

D7 **(a)** $4x+5y=40$ and $3x+y=18$: $\boxed{(\frac{50}{11},\frac{48}{11}),\ P=\frac{442}{11}}$

(b) $(4,4)$: 36; $(5,3)$: 37; $(5,4)$ and $(6,2)$ not in R: $\boxed{(5,3),\ P=37}$

D8 **(a)** $\boxed{(0,0),\ (0,6),\ (6,4),\ (7,2),\ (7,0)}$

(b) edges at $(6,4)$ have gradients $-\frac{1}{3}$ and -2; the objective line has gradient $-\frac{k}{3}$: $-2\leqslant -\frac{k}{3}\leqslant -\frac{1}{3}$: $\boxed{1\leqslant k\leqslant 6}$

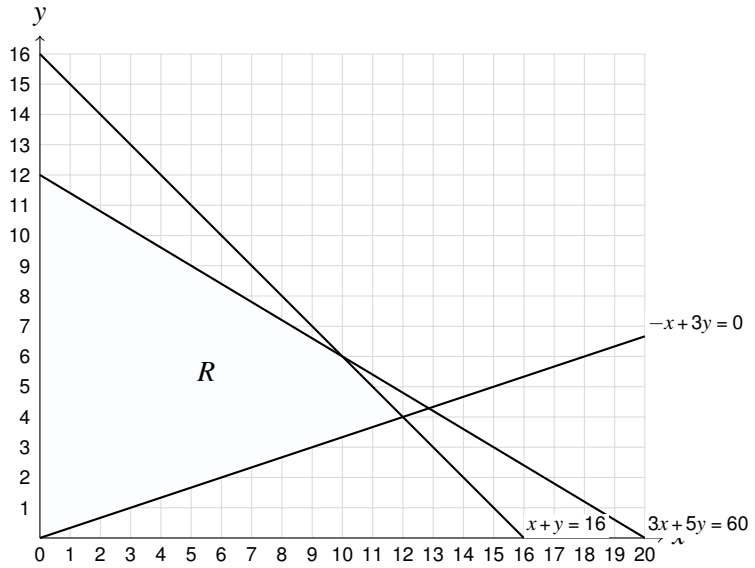
E Exam practice solutions

Mark-scheme notes are shown on the right of each line.

CORE *Exam questions for everyone*

E1 **(a)** $3x+5y \leqslant 60$; $15x+15y \leqslant 240$: $x+y \leqslant 16$; $x \leqslant 3y$ *B1 B1 B1*

(b)



Solution to E1(b)

B1 B1 B1 B1

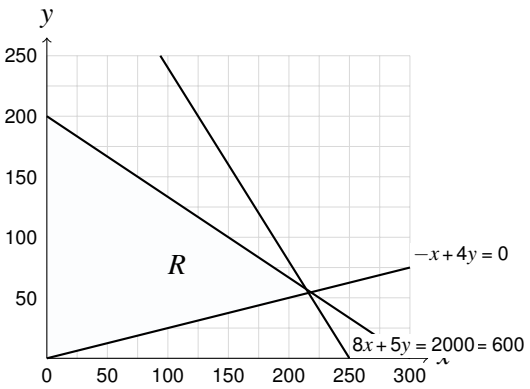
(c) objective line $5x+6y=k$ drawn; last point of R reached is $(10,6)$ *M1 A1*

10 small and 6 large bouquets, £86 *A1*

E2 **(a)** $\frac{x}{300}+\frac{y}{200}\leqslant 1$ gives $2x+3y\leqslant 600$; $\frac{x}{250}+\frac{y}{400}\leqslant 1$ gives $8x+5y\leqslant 2000$ *M1 A1*

$y\geqslant 0.2(x+y)$: $\boxed{x\leqslant 4y}$ *B1*

(b)



Solution to E2(b)

B1 B1 B1

(c) vertices $(0,0)$, $(0,200)$, $(\frac{1500}{7},\frac{400}{7})$, $(\frac{8000}{37},\frac{2000}{37})$; P: 0, 1200, $\frac{9900}{7}$, $\frac{52000}{37}$ *M1 A1*

$\boxed{(\frac{1500}{7},\frac{400}{7})}$**, £**$\frac{9900}{7}$ **(£1414.29)** *A1*

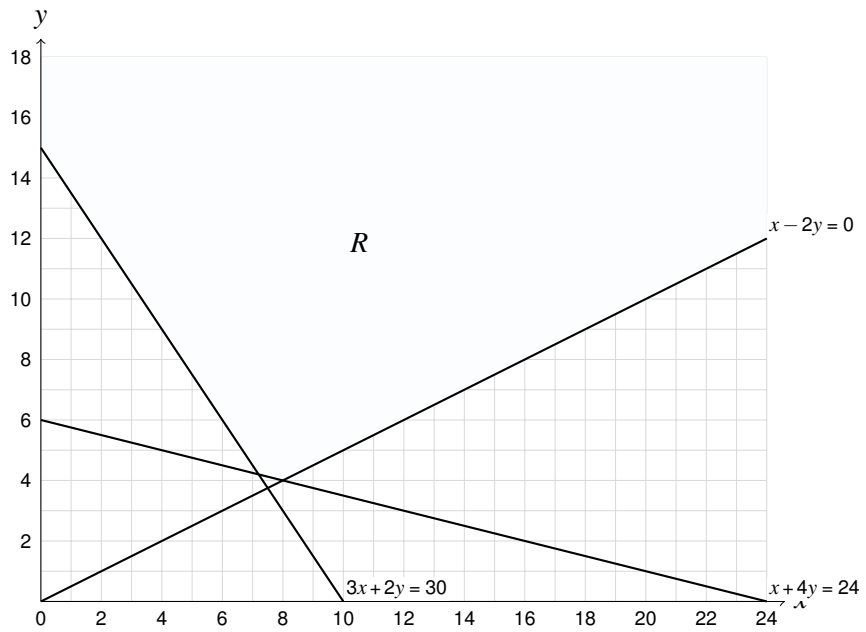
(d) $(214,57)$: 1412; $(213,58)$: 1413; $(215,56)$: 1411; $(214,58)$ breaks $2x+3y\leqslant 600$ *M1 A1*

$\boxed{\textbf{213 of } A \textbf{ and 58 of } B\textbf{, £1413}}$ *A1*

STRETCH

E3 **(a)** **Minimise $C = 5x + 4y$**, subject to $3x + 2y \geqslant 30$, $x + 4y \geqslant 24$, $x \leqslant 2y$, $x,\ y \geqslant 0$ *B1 B1 B1*

(b)



Solution to E3(b) (R is unbounded)

B1 B1 B1

(c) objective line moved towards the origin; first point of R reached is where $3x + 2y = 30$ meets $x + 4y = 24$ *M1*

$(\frac{36}{5}, \frac{21}{5})$, £$\frac{264}{5}$ = £52.80 *A1 A1*

(d) (7,4), (6,5) and (7,5) are tested: (7,4) and (6,5) are not in R; (7,5): 55, (6,6): 54, (8,4): 56 *M1*

6 bags of A and 6 of B, £54 *A1*

(e) edges at the vertex have gradients $-\frac{3}{2}$ and $-\frac{1}{4}$; the objective line has gradient $-\frac{c}{4}$ *M1 A1*

$-\frac{3}{2} \leqslant -\frac{c}{4} \leqslant -\frac{1}{4}$: $1 \leqslant c \leqslant 6$ *A1*