



eClassroom

A-LEVEL MATHEMATICS • PURE 1

# Differentiation

Worked Solutions

Pearson Edexcel International A Level (WMA11) & A Level (9MA0) • Calculator allowed: show every step

Full worked solutions for the free Differentiation worksheet, A-Level and International A-Level Maths



Tap a section to jump straight to it

**Full worked solutions** for every question on the Pure 1 Differentiation worksheet, in the same order and with the same question numbers.

**How to mark your work.** Tick each step you got right, not just the final answer. At A-level most marks are for method, so a wrong final answer can still earn most of the marks. If you lost marks, find the *first* line where your working is different from ours. That is the thing to fix.

In Section E: **M** = method mark, **dM** = method mark that depends on the previous M mark, **A** = accuracy mark (needs the M mark before it), **B** = independent mark.

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**A****Your turn answers**

Compare each line of your working with the solution.

**CORE**

**A1**  $y = 2x^5 + 3x^{-1} - 8x^{\frac{1}{2}}$ :  $\frac{dy}{dx} = 10x^4 - 3x^{-2} - 4x^{-\frac{1}{2}}$

**A2**  $\frac{dy}{dx} = 4x - 8x^{-2}$ ; at  $x = 2$ :  $8 - 2 = 6$

**A3** Point:  $y = -1 + 2 = 1$ . Gradient:  $3x^2 - 2 = 1$ .  $y - 1 = 1(x + 1)$ :  $y = x + 2$

**STRETCH**

**A4**  $y = \frac{1}{2}x^{\frac{3}{2}} - \frac{5}{2}x^{\frac{1}{2}}$ :  $\frac{dy}{dx} = \frac{3}{4}x^{\frac{1}{2}} - \frac{5}{4}x^{-\frac{1}{2}}$

**A5**  $\frac{dy}{dx} = x^{-\frac{1}{2}} = \frac{1}{2}$  at  $x = 4$ , so the normal has gradient  $-2$ :  $y - 4 = -2(x - 4)$ :  $2x + y - 12 = 0$

**A6**  $f'(x) = 6x^2 - 8x^{-2}$ ,  $f''(x) = 12x + 16x^{-3}$ :  $f''(2) = 24 + 2 = 26$

**A7**  $3x^2 + 6x - 7 = 2$ , so  $x^2 + 2x - 3 = 0$ ,  $(x + 3)(x - 1) = 0$ ;  $y = x^3 + 3x^2 - 7x$ :  $(1, -3)$ ,  $(-3, 21)$



## B

## Fluency solutions

One mark for each correct part. The notes in *italics* explain why each sequence was chosen.

## CORE

B1 (a)  $6x^5$  (b)  $-10x^{-3}$  (c)  $6x^{\frac{1}{2}}$  (d)  $-2x^{-\frac{4}{3}}$

Multiply by the power, then take 1 off the power, even when it is negative or a fraction.

B2 (a)  $\frac{1}{2}x^{-\frac{1}{2}}$  (b)  $-3x^{-2}$  (c)  $2x^{-\frac{1}{2}}: -x^{-\frac{3}{2}}$  (d)  $x^{\frac{5}{2}}: \frac{5}{2}x^{\frac{3}{2}}$

Roots become fractional powers, and  $x$  on the bottom becomes a negative power.

B3  $\frac{dy}{dx} = 3x^2 - 12$  (a)  $-12$  (b)  $0$  (c)  $0$  (d)  $15$

$3x^2$  is the same for  $x$  and  $-x$ , so (b) and (c) give the same gradient.

B4 (a) (1, 1), gradient 2:  $y = 2x - 1$  (b) (1, 4), gradient 2:  $y = 2x + 2$

(c) (1, 3), gradient 6:  $y = 6x - 3$  (d) (1, 1), gradient 3:  $y = 3x - 2$

Moving the curve up keeps the gradient but moves the point; stretching it changes both.

B5 (a) gradient  $-\frac{1}{2}$  through (1, 1):  $x + 2y - 3 = 0$  (b)  $-\frac{1}{3}$  through (1, 1):  $x + 3y - 4 = 0$

(c)  $\frac{dy}{dx} = -2x^{-2} = -2$ , so the normal has gradient  $\frac{1}{2}$  through (1, 2):  $x - 2y + 3 = 0$

The normal's gradient is  $-\frac{1}{m}$ ; a negative tangent gradient gives a positive normal gradient.

B6 (a)  $12x^2$  (b)  $2x^{-3}$  (c)  $-\frac{1}{4}x^{-\frac{3}{2}}$  (d)  $3x^{-\frac{1}{2}}$

Differentiate twice; take care with the signs of negative powers.

## STRETCH

B7  $f'(x) = 3x^2 - 3$  (a)  $x = \pm 1$  (b)  $3x^2 = 12: x = \pm 2$  (c)  $x = 0$  (d)  $3x^2 = -3: \text{no values}$

The least value of  $3x^2 - 3$  is  $-3$ , so a gradient of  $-6$  never happens.



**B8** (a)  $\frac{27-8}{1} = 19$  (b)  $\frac{2 \cdot 1^3 - 8}{0.1} = 12.61$

(c)  $\frac{(2+h)^3 - 8}{h} = \frac{12h + 6h^2 + h^3}{h} = 12 + 6h + h^2$  **as required**

(d) As  $h \rightarrow 0$ : **12**

The chords get closer to the tangent as  $h$  shrinks: this is the derivative from first principles.

**B9** (a)  $3kx^2 = 12k = 12$ :  **$k = 1$**  (b)  $2x + k = 6 + k = 0$ :  **$k = -6$**  (c)  $-kx^{-2} = -\frac{k}{4} = -2$ :  **$k = 8$**  (d)  $\frac{k}{2\sqrt{x}} = \frac{k}{4} = 3$ :  **$k = 12$**

Differentiate with  $k$  in place, then substitute the value of  $x$ .

**B10**  $f'(x) = 3x^2 - 12x$ ,  $f''(x) = 6x - 12$

(a)  **$-6$**  (b)  **$x = 2$**  (c)  $f'(2) = 12 - 24 = -12$

(d)  $3x^2 - 12x = 6x - 12$ , so  $x^2 - 6x + 4 = 0$ :  **$x = 3 \pm \sqrt{5}$**

Keep  $f'$  and  $f''$  separate: (c) asks for  $f'$  at the point found from  $f''$

## C

## Reasoning solutions

Your wording can be different. Check that your reason makes the same point.

## CORE

**C1** Sam differentiated the bottom of the fraction. Write it as a power first:  $y = 3x^{-2}$ , so  $\frac{dy}{dx} = -6x^{-3}$

**C2** You cannot differentiate a fraction by differentiating the top and the bottom. Divide first:  $y = x + x^{-1}$ , so  $\frac{dy}{dx} = 1 - x^{-2}$

**C3**  $3x^2 - 6x = -3$ , so  $x^2 - 2x + 1 = 0$ ,  $(x - 1)^2 = 0$ : only  $x = 1$ , a repeated root.

At  $x = 1$ ,  $y = 3$ :  $y - 3 = -3(x - 1)$ :  **$y = -3x + 6$**

**C4** Kai used the gradient of the tangent. The normal is perpendicular, so its gradient is  $-\frac{1}{10}$

$y - 4 = -\frac{1}{10}(x - 2)$ , so  $10y - 40 = -x + 2$ :  **$x + 10y - 42 = 0$**



## STRETCH

**C5** A parallel tangent has the same *gradient*, not the same line, so Jo should solve  $f'(x) = f'(2)$

$$f'(x) = 3x^2 - 3 \text{ and } f'(2) = 9: 3x^2 - 3 = 9, x = \pm 2; \text{ the other point is at } x = -2, y = -8 + 6: \boxed{(-2, -2)}$$

**C6** Gradient of the chord from  $x$  to  $x+h$ :  $\frac{(x+h)^3 - x^3}{h} = \frac{3x^2h + 3xh^2 + h^3}{h} = 3x^2 + 3xh + h^2$

As  $h \rightarrow 0$ , this tends to  $3x^2$ , so the derivative of  $x^3$  is  $3x^2$  **as required**

**C7**  $\frac{dy}{dx} = 3x^2 + 3$ . Gradient 2 needs  $3x^2 = -1$ , which is impossible because  $x^2 \geq 0$

$3x^2 + 3 \geq 3$ , with equality at  $x = 0$ : least gradient **3**

**C8** (a)  $\frac{dy}{dx} = 3x^2$ : at  $x = a$  and  $x = -a$  the gradient is  $3a^2$ , so the tangents are parallel.

(b) Point  $(a, a^3)$ :  $y - a^3 = 3a^2(x - a)$ , so  $y = 3a^2x - 2a^3$  **as required**

(c) The tangents cross the  $y$ -axis at  $-2a^3$  and  $2a^3$ :  $4a^3 = 4$ , so  $\boxed{a = 1}$ .  $\frac{d^2y}{dx^2} = 6x$ : **6** at  $x = 1$  and **-6** at  $x = -1$

## D

## Problem solving solutions

There is often more than one correct method. Any complete, correct method scores full marks.

## CORE

**D1** (a)  $y = 27 - 45 + 12 + 7 = 1$ ;  $\frac{dy}{dx} = 3x^2 - 10x + 4 = 1$  at  $x = 3$

$$y - 1 = 1(x - 3): \boxed{y = x - 2}$$

(b)  $y = 0$ :  $x = 2$ , and  $y = x - 2$ : **(2, 0)**

**D2** (a)  $y = x^{\frac{3}{2}} - 6x^{\frac{1}{2}}$ :  $\frac{dy}{dx} = \frac{3}{2}x^{\frac{1}{2}} - 3x^{-\frac{1}{2}}$

(b) At  $x = 4$ :  $y = 8 - 12 = -4$ , gradient  $3 - \frac{3}{2} = \frac{3}{2}$ , so the normal has gradient  $-\frac{2}{3}$

$$y + 4 = -\frac{2}{3}(x - 4), \text{ so } 3y + 12 = -2x + 8: \boxed{2x + 3y + 4 = 0}$$



**D3** (a)  $\frac{dy}{dx} = 2kx + 6x^{-2}$ ; at  $x = 1$ :  $2k + 6 = 10$ :  $k = 2$

(b) At  $x = -1$ :  $-4 + 6 = 2$

**D4** (a)  $f'(x) = 6x^2 - 18x + 12 = 6(x-1)(x-2) = 0$ :  $x = 1$ ,  $x = 2$

(b)  $f''(x) = 12x - 18$ :  $f''(1) = -6$ ,  $f''(2) = 6$

## STRETCH

**D5** (a)  $\frac{dy}{dx} = 1 - 9x^{-2} = -8$ , so  $9x^{-2} = 9$ , so  $x^2 = 1$  **as required**

(b)  $x = 1$ :  $y = 10$ , tangent  $y - 10 = -8(x - 1)$ :  $y = -8x + 18$

$x = -1$ :  $y = -10$ , tangent  $y + 10 = -8(x + 1)$ :  $y = -8x - 18$

**D6** (a)  $\frac{dy}{dx} = -4x^{-2} = -4$  at  $x = 1$ :  $y - 4 = -4(x - 1)$ , so  $y = -4x + 8$ .  $A(2, 0)$ ,  $B(0, 8)$ : area **8**

(b) At  $\left(a, \frac{4}{a}\right)$  the gradient is  $-\frac{4}{a^2}$ :  $y - \frac{4}{a} = -\frac{4}{a^2}(x - a)$ , so  $y = -\frac{4}{a^2}x + \frac{8}{a}$

It meets the axes at  $(2a, 0)$  and  $\left(0, \frac{8}{a}\right)$ : area  $= \frac{1}{2} \times 2a \times \frac{8}{a} = 8$  for every  $a$  **as required**

**D7** (a)  $\frac{dy}{dx} = 2x - 4 = 2$  at  $x = 3$ , so the normal has gradient  $-\frac{1}{2}$ :  $y = -\frac{1}{2}(x - 3)$ , so  $x + 2y - 3 = 0$

(b)  $x^2 - 4x + 3 = -\frac{1}{2}(x - 3)$ , so  $2x^2 - 7x + 3 = 0$ ,  $(2x - 1)(x - 3) = 0$ ;  $x = 3$  is  $P$

$x = \frac{1}{2}$  and  $y = x^2 - 4x + 3$ :  $Q\left(\frac{1}{2}, \frac{5}{4}\right)$

**D8** (a)  $f(2) = 4a + \frac{b}{2} = 10$ , so  $8a + b = 20$ ;  $f'(x) = 2ax - bx^{-2}$ ,  $f'(2) = 4a - \frac{b}{4} = 7$ , so  $16a - b = 28$  **as required**

(b) Adding:  $24a = 48$ :  $a = 2$ ,  $b = 4$

(c)  $f''(x) = 2a + 2bx^{-3} = 4 + 8x^{-3}$ :  $f''(2) = 4 + 1 = 5$



## E

## Exam practice solutions

Mark-scheme notes are shown on the right of each line.

## CORE

Exam questions for everyone

E1

(a)  $\frac{dy}{dx} = 3x^{\frac{1}{2}} - 3$

M1: one power reduced; A1

(b) At  $x = 4$ :  $y = 16 - 12 + 5 = 9$ , and  $\frac{dy}{dx} = 6 - 3 = 3$

M1: substitutes  $x = 4$  into  $\frac{dy}{dx}$ 

$$y - 9 = 3(x - 4)$$

dM1

$$3x - y - 3 = 0$$

A1: integers, = 0

E2

(a) (i)  $f'(x) = 4x - 8x^{-2}$

(ii)  $f''(x) = 4 + 16x^{-3}$

B1 B1ft

(b)  $f(2) = 8 + 4 = 12$  and  $f'(2) = 8 - 2 = 6$ , so the normal has gradient  $-\frac{1}{6}$

M1:  $-\frac{1}{f'(2)}$ 

$$y - 12 = -\frac{1}{6}(x - 2)$$

dM1

$$x + 6y - 74 = 0$$

A1

(c)  $4 + \frac{16}{8} = 6$

B1ft

## STRETCH

E3

(a)  $\frac{dy}{dx} = \frac{1}{2}x = p$  at  $x = 2p$

M1: differentiates

$$A \text{ is } (2p, p^2): y - p^2 = p(x - 2p)$$

dM1

$$y = px - p^2$$

as required

A1\*

(b) The gradients are  $p$  and  $q$ ; perpendicular when  $p \times q = -1$

as required

M1 A1\*

(c)  $px - p^2 = qx - q^2$ , so  $(p - q)x = p^2 - q^2 = (p - q)(p + q)$

M1: equates the tangents

$$p \neq q, \text{ so } x = p + q, \text{ and } y = p(p + q) - p^2 = pq$$

dM1 A1

$$pq = -1, \text{ so } T \text{ lies on } y = -1$$

as required

